Market for votes

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Abstract

We construct a centralised market where agents can trade voting rights. Text book markets do not work but the situation is strategic, and there are many equilibria. In an equilibrium where trading takes place the minimun efficiency gain, i.e. the probability with which an efficient outcome is chosen, is between 12% and 37% depending on the number of voters.

1 Introduction

Voting is not necessarily efficient. An easy way to see this is to consider the following scenario where there are two types of people whom we call A-type and B-type. The number of the former is a and the number of the latter is b. A-type agents support candidate A and get utility α if he is elected. B-type agents support candidate B and get utility β if he is elected. If an agent's favourite candidate is not elected he gets utility zero.

Let us assume that $a > b$ which means that candidate A is elected in a majority voting. If, however, $a\alpha < b\beta$ the outcome is not efficient. In voting situations each agent is normally endowed with one vote, and one may raise the question why there is not a market for votes. Of course, in many countries and situations vote trading is prohibited by law but so are many other things, and still there exist markets for them.

Before analysing the market for votes let us note that even if monetary transfers are possible the core of this economy is empty. Whenever the votes are distributed in such a way that candidate A is elected the grand coalition can improve and reach the efficient solution. Assume that the minimun number of votes to tip the result of the election in favour of candidate B is k. If the votes are distributed in such a way that candidate B is elected, and the B-type agents acquire more than k votes (using monetary transfers) an improving coalition can be formed by throwing one type A agent away. Finally, if the B-type agents buy exactly k votes some A-type agents do not sell their vote and they can offer their vote for sale at a lower price, i.e., any one of them can propose an improving coalition. In light of this result the possibility of establishing a market looks somewhat dismal.

Let us anyway see how one could proceed to put up a market for voting rights. The standard Walrasian market of an economics text book is characterised by a uniform price (for a uniform good), and in equilibrium supply equals demand. Assume that the price for voting rights is π. It is clear that no A-type agent wants to buy any votes since they already have sufficient votes to get their favourite candidate elected. Here we have assumed that in equilibrium it is not the case that agents of the same type may end up both as buyers and sellers. But this is clear as can be seen below.

Assume that amongst the A-type agents some buy and some sell. Also assume that the final outcome is that candidate A is elected. Since in equilibrium as many votes are sold as bought, and even more strictly as many votes are offered for sale as are demanded, at least one A-type agent gets at most utility $\alpha - \pi$ which must be positive. Now, it is not possible that any B-type agent buys a vote because he would get utility $-\pi$. But now the A-type agent who bought the vote of another A-type agent could improve his utility by not buying the vote which would remain at the original owner.

So, assume that there are buyers and sellers amongst the B-type agents. This makes sense only if the outcome is B. As before no A-type agent can be a buyer in equilibrium. If there are to be buyers and sellers amongst the B-type agents it must be the case that in equilibrium the B-type agents buy from A-type agents so many votes that the votes B-type agents own exceed the votes owned by A-type agents by one or two. Then the B-type agent who buys the vote of another B-type agent could improve his utility by refraining. Further, in anonymous markets all the A-type agents would like to sell their votes at a positive price if they expect that candidate B is elected.

The above analysis proceeds along the lines of anonymous Walrasian markets even though it is clear that vote trading involves aspects of co-ordination and free riding. Totally anonymous markets seem not possible since the agents must have expectations about the outcome and consequently about the number of votes offered for sale and demanded. Also, all agents would like to free ride and let others of their type to buy the votes. One can construct equilibria where all agents of one type offer their votes for sale and a particular agent or particular agents of the opposite type are assumed to buy the votes. But that can take place only in specific cases. In our setting, for instance, one could number the B-type agents from 1 to b and postulate that sufficiently many of them, in increasing order, buy a vote. The sufficient number would just tip the number of votes in B-type's advantage. But, of course this number is less than a and in equilibrium supply could not equal demand.

The analysis so far strongly hints towards the fact that in constructing Walrasian type markets one must take into consideration some kind of strategic aspects, i.e. the situation is game like. Further, it looks like that there only exists a mixed stratetgy equilibrium if one wants to consider symmetric strategies which is in line with the anonymity assumption of Walrasian markets. This is what we do in the next section. In section 3 we consider efficiency questions.

Before proceeding with the analysis we mention related literature. Surprisingly, there is not much. Even though the problems of free riding and coordination are well known in this setting and logrolling and implicit vote selling has been analysed in committees (Strattman, 1992) studies about centralised markets for voting rights are few. Closest to our article is Weiss (1988) where voters decide on the amount of taxation that turns into a public good. The voters differ as to their preferences and endowments. His main result is that generically the core of the vote trading game is empty. Neeman (1999) analyses the free rider problem in general and applies his results to a vote trading situation where one large voter aims at buying the votes of small voters (magnitude in the sense of achievable utilities). He shows that the large voter can effect an inefficient outcome.

2 Centralised markets

Here the aim is to construct an institution that emulates the Walrasian markets as closely as possible. One thing that we want to retain is anonymity, and to this end we consider equilibria only in mixed strategies. Let us first give the definition of equilibrium.

Definition 1 *An equilibrium is a price for votes π, the probability of selling (buying) a vote m (n) for A-types and the probability of buying (selling) a vote p (q) for B-types, such that the agents' strategies maximise their expected utility given their beliefs.*

We assume that $a > b$, and that there is an odd number of agents in the economy. Then, regardless of the number of votes that are bought either A or B wins as there are no draws. Then the minimun number of votes needed to tip the election in favour of candidate B is $k = \left[\frac{a-b}{2} + 1\right]$ where the brackets signify the greatest integer function. Let us assume that the market price for votes is π. Now we guess, and later confirm, that only B-type agents buy votes. The probability with which they buy a single vote is p. Before proceeding we should make clear the procedure of the centralised market. There are many possibilities of which we focus on the following. The agents have expectations about who will sell and who will buy votes. The organiser of the centralised markets has an objective function that is zero if an inefficient outcome results, and unity otherwise. Thus, he announces the market price π which the agents observe such that at least in theory it could generate trade in equilibrium. Then the agents submit supplies and demands. In this setting there are, of course, equilibria in which no trading takes place. For instance, if everyone expects that no A-type agent sells his vote no B-type agent is willing to buy.

Next we construct an equilibrium in which all A-type agents offer their votes for sale and the B-type agents buy votes with probability p. To shorten the notation let us denote the distribution function of a binomial $(b-1,p)$ by F, so that $F(i)=\sum_{j=0}^{i}\binom{b-1}{j}p^j(1-p)^{b-1-j}$. To figure out the equilibrium mixed strategy is easy; we just have to make sure that a buyer, i.e., a B-type agent, is indifferent between buying and not buying. If he buys a vote his utility is

$$F(k-2)(0-\pi)+(1-F(k-2))\,(\beta-\pi) \tag{1}$$

If he does not buy a vote his utility is

$$(1-F(k-1))\,\beta \tag{2}$$

Equating (1) and (2) yields the following condition that determines p

$$\pi=\binom{b-1}{k-1}p^{k-1}(1-p)^{b-k}\beta \tag{3}$$

It is immediate that when thought of as a function of p the price is maximised at $p=\frac{k-1}{b-1}$.

For this setting to be an equilibrium we must make sure that no-one wants to deviate. For sellers this is clear; since there are more A-type agents than B-type agents no seller can affect anything by not offering his vote for sale. The only thing that is affected is that the deviating agent forfeits the chance of receiving π. This also shows that when the agents expect k or more votes to be offered for sale then all the A-type agents want to offer their votes for sale. And B-type agents submit any bids for votes only if they expect k or more votes to be offered for sale Next we make sure that a B-type agent does not find it profitable to become a seller. If he offers his vote for sale his expected utility is

$$\begin{array}{c}\sum_{j=0}^{k-1}\binom{b-1}{j}p^j(1-p)^{b-1-j}\frac{j}{a+1}\pi+\\ \sum_{j=k+1}^{b-1}\binom{b-1}{j}p^j(1-p)^{b-1-j}\left(\frac{j}{a+1}\pi+\beta\right)+\\ \binom{b-1}{k}p^k(1-p)^{b-1-k}\left(\frac{k}{a+1}\pi+\frac{a+1-k}{a+1}\beta\right)\end{array} \tag{4}$$

$$\binom{b-1}{k}p^k(1-p)^{b-1-k}\left(\frac{k}{a+1}\pi+\frac{a+1-k}{a+1}\beta\right)$$

A little manipulative calculation shows that (4) < (2) iff and only iff

$$\sum_{j=0}^{b-1}\binom{b-1}{j}p^j(1-p)^{b-1-j}\frac{j}{a+1}\pi<\binom{b-1}{k}p^k(1-p)^{b-1-k}\frac{k}{a+1}\beta \tag{5}$$

Inserting the equilibrium condition (3) into (5) and manipulating a little yields the following condition

$$\frac{b-2}{b-1}<p \tag{6}$$

The left hand side of (6) is bigger than the value of p that maximises the price of the vote, and for $b\geq 3$ the left hand side of (6) is at least one half.

This means that there always exists a price π such that our construction is an equilibrium. Of course, there exist many such prices the greatest one being the one resulting from choice $p = \frac{b-2}{b-1}$. Notice that for all prices but $p = \frac{k-1}{b-1}$ there are two values of p that satisfy equation (3) but condition (6) selects one of them.

We also must make certain that a B-type agent does not want to buy 2 votes. This is the case if $\pi > \binom{b-1}{k-2} p^{k-2}(1-p)^{b-k+1}\beta$. Inserting the equilibrium condition (3) here we get condition $p > \frac{k-1}{b}$.

The interesting thing is the efficiency of this equilibrium but before that let us study the deviation possibilities of A-type agents. It is imaginable that an A-type agent observing price π notices that he is not certain to get this price. But he could approach a B-type agent directly and offer his vote for sale at some price $\pi' < \pi$. The B-type agent would certainly agree since he is indifferent between buying at π and not buying at all. To make sure that this verbal reasoning is correct let us first calculate the A-type agent's utility in equilibrium.

$$\begin{gathered} \sum_{j=0}^{b} \binom{b}{j} p^j (1-p)^{b-j} \tfrac{j}{a}\pi + \sum_{j=0}^{k-1} \binom{b-1}{j} p^j (1-p)^{b-j}\alpha = \\ p\tfrac{b}{a}\pi + \sum_{j=0}^{k-1} \binom{b-1}{j} p^j (1-p)^{b-j}\alpha \end{gathered} \tag{7}$$

Selling his vote at price π' to a B-type agent yields

$$\pi' + \sum_{j=0}^{k-2} \binom{b-1}{j} p^j (1-p)^{b-1-j}\alpha \tag{8}$$

Now (7) is greater than (8) if and only if

$$\pi \left(p\frac{b}{a} + (1-p)\frac{\alpha}{\beta} \right) > \pi' \tag{9}$$

The coefficient of π in (9) is less than unity, and consequently the exist π' that are greater than the left hand side of (9) and less than π. So, only the lowest possible equilibrium price $\pi = 0$ is immune against this kind of deviation but then $p = 1$. For a centralised market to function at all it must be the case that this kind of deviation is not feasible. This may be the case if vote trading is prohibited outside the market place or people do not know each others' identity.

3 Efficiency

On the basis of the above analysis the key to establishing functioning markets for votes in this setting is not to allow private exchange. Next we study what is the minimum efficiency gain in an equilibrium in which trading takes place. This is achieved when the price of a vote is at its maximum which is $\pi = \frac{b-2}{b-1}$.

Of course, the probability of buying a vote is at its minimum consistent with (6).

We only consider cases where the B-type agents are so numerous that in equilibrium they buy at most one vote. This means that at maximum $k = \left[\frac{a-b}{2} + 1\right]$ can equal b. This is equivalent to $a = 3b - 1$. In this case B-type agents have to buy B votes, i.e. one vote each in order to get their favourite elected. This happens with probability

$$p^b = \left(\frac{b-2}{b-1}\right)^b = \left(1 - \frac{1}{b-1}\right)^b \tag{10}$$

The magnitude in (10) increases towards $e^{-1} \approx 0.367$ when b grows without limit. Already when $b = 3$ (10) equals 0.125. This way we get an idea about the minimum efficiency that one can expect in this kind of voting right markets in an equilibrium where the agents trade. In a very large society the probability that the efficient outcome, i.e. B, comes about is more than 35%. This may not be a particularly impressive figure. In particular, if one takes into account that there are necessarily other equilibria where no trade takes place, and the inefficient outcome is certain.

4 Conclusion

We have strived at constructing a centralised market for voting rights. One may think of this as a futile enterprise, but we think that the features that are often mentioned as reasons for the lack of such markets are nicely illuminated here. Free riding and co-ordination problems preclude text book centralised market, and the best we can get is a market where the agents' behaviour is strategic and where multiple equilibria exist. On top of that, it is necessary to prevent private vote trading; all trading must take place via the centralised markets.

The probability of the efficient outcome is at the minimum between 12% and 37% depending on the number of the agents. This kind of efficiency gain would probably be thought of remarkable in other sectors of the economy. If one believes in the co-ordinating ability of the government these gains could also be achievable without immense problems, as the government could choose the equilibrium where trading takes place.

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